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LAMINAR FILM CONDENSATION IN TUBES; CALCULATION OF LOCAL FILM RESISTANCE AND LOCAL ADIABATIC MIXING TEMPERATURE

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Abstract—For combined gravity and shear forces between turbulent vapour and laminar condensate (co-current or counter-current flow) the local thickness of the film and its adiabatic mixing temperature were calculated.

For the accurate calculation of the film thickness the condensate viscosity profile was taken into account, resulting in three different reference temperatures for the viscosity.

For the calculation of the local adiabatic mixing temperature constant local condensate properties and a linear condensate temperature profile was assumed. Then, when taking gravity *and* shear forces into account the local mixing temperature is no longer constant (for given local wall and film surface temperatures) but a function of many variables.

From the analytical (though iterative) solution approximating equations were derived for the local film resistance, facilitating calculation "by hand".

NOMENCLATURE

A, dimensionless group defined by (29);
a, fixed numerical value, see (22);
D, inside tube diameter;
f, variable defined by (19);
Gr, Grashof number defined by (11);
g, gravitational acceleration;
h, variable defined by (19);
M, dimensionless adiabatic mixing temperature defined by (33);
$\dot{m}$, mass flow rate;
N, dimensionless group defined by (37) or (38);
Nu, Nusselt number defined by (12);
p, pressure;
Q, dimensionless group defined by (35);
Re, Reynolds number;
T, temperature of the condensate;
w, velocity;
x, co-ordinate in vapour flow direction;
y, co-ordinate in heat flux direction (normal to the wall);
β, angle between vapour flow direction (x-co-ordinate): and gravitational acceleration (vector g), see Fig. 1;
δ, thickness of the condensate film, see Fig. 2;
ζ, dimensionless pressure drop defined by (7);
ν, kinematic viscosity;
ξ, dimensionless co-ordinate in heat flux (y) direction (normal to the wall) defined by (2);
ρ, density.

Subscripts

C, critical (laminar–turbulent);
g, only gravitational effects (no shear forces of the vapour);
L, of the liquid or condensate;
S, at the condensate film surface;
V, of the vapour (properties at its bulk temperature), or with Nu caused only

by vapour shear forces (no gravitational effects);

V, L, determined with vapour difference velocity and liquid kinematic viscosity;

W, at the wall;

v, for the determination of $\bar{v}$;

ξ, as a function of ξ;

ξ, $\frac{1}{2}, \frac{2}{3}, \frac{3}{4}$ determined with the viscosities at these points.

Superscripts and bars

$-$, mean value.

1. INTRODUCTION

FOR THE *incremental* calculation of isothermal and non-isothermal condensers one needs the *local* condensation heat transfer coefficient. The condensate is formed completely or partly at the cooled wall, frequently as a laminar film. In air-cooled condensers which are applied frequently nowadays, the vapour condenses *inside* tubes and in many cases the shear forces of the vapour cannot be neglected.

Nusselt [1] extended his own theory to include the vapour shear effect. He and a few later authors [2, 3] considered the laminar film condensation of a single vapour of a nearly constant wall temperature and developed an area average heat transfer coefficient.

Carpenter and Colburn [4] and after them several others referred to by Narayana Murthy [5] consider film condensation inside tubes, but only in the range of *turbulent* film flow and neglecting gravity forces.

The current paper covers the range of *laminar* film condensation. Equations are developed for the calculation of the local heat transfer resistance of the film and its local adiabatic mixing temperature, assuming a linear temperature profile in the film. This assumption provides a good approximation on the "safe" side with respect to heat transfer. Shear and gravity forces were taken into account. The condensate viscosity profile was incorporated in the calculation of the condensate film thickness in any cross section, where the other properties are considered constant.

2. THE BALANCE OF FORCES

Consider a tube inclined at an angle β, compared with the direction of gravitational (or other) acceleration, as shown in Figs. 1 and 2.

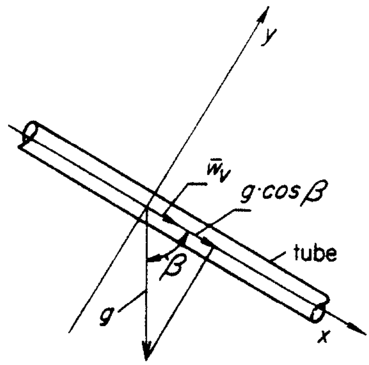

FIG. 1. Vapour carrying tube in the gravity field.

Like Nusselt we make the following reasonable and simplifying assumptions:

(i) laminar waveless film flow along the tube wall only parallel to the tube axis, with uniform film thickness along any circumference

(ii) that acceleration forces can be neglected

(iii) a very thin film compared to the inside tube diameter ($\delta \ll D$).

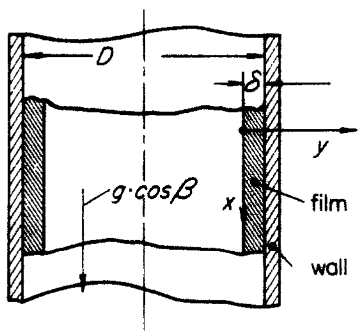

FIG. 2. Condensate film inside the tube.

The first two assumptions are on the "safe" side: if not fulfilled the actual heat-transfer resistance is lower than the calculated one. The same applies to assumption (iii) for the usual case of co-current vapour and condensate flow.

Equilibrium of the forces over the length dx and after dividing by $D.\pi.\mathrm{d}x$ (see also [6], (1)) gives:

$$y.(\rho_L - \rho_V).g.\cos\beta - \frac{D}{4}\cdot\frac{\mathrm{d}p}{\mathrm{d}x} = -\nu_L\cdot\rho_L\cdot\frac{\mathrm{d}w_L}{\mathrm{d}y}. \quad (1)$$

As in a previous paper [6] we introduce the dimensionless variable

$$\xi = \frac{y}{\delta} \quad (2)$$

and the variable viscosity

$$\frac{1}{\nu_{L,\xi}} = \frac{1}{\nu_S} + \xi\cdot\left(\frac{1}{\nu_W} - \frac{1}{\nu_S}\right). \quad (3)$$

Then, with the boundary condition: $w_L = 0$ for $\xi = 1$, we obtain by integration of (1) the velocity as function of ξ:

$$w_L = \frac{\delta^2}{2}\cdot\frac{\rho_L - \rho_V}{\rho_L}.g.\cos\beta\left[\frac{1}{\nu_S}\cdot(1-\xi^2) + \frac{2}{3}\left(\frac{1}{\nu_W} - \frac{1}{\nu_S}\right)(1-\xi^3)\right] - \frac{D.\delta}{4.\rho_L}\cdot\frac{\mathrm{d}p}{\mathrm{d}x} \times\left[\frac{1}{\nu_S}(1-\xi) + \frac{1}{2}\left(\frac{1}{\nu_W} - \frac{1}{\nu_S}\right)(1-\xi^2)\right] \quad (4)$$

with $\xi = 0$ we find the velocity of the condensate film surface

$$w_S = \frac{\delta^2}{2.\nu_{L,\xi=\frac{1}{3}}}\cdot\frac{\rho_L - \rho_V}{\rho_L}.g.\cos\beta - \frac{D.\delta}{4\rho_L.\nu_{L,\xi=\frac{1}{2}}}\cdot\frac{\mathrm{d}p}{\mathrm{d}x}. \quad (5)$$

The subscript $\xi = \ldots$ means that the viscosity has to be determined according to (3) or, as shown in [6], at the reference temperature $T_\nu = \xi.T_W + (1-\xi).T_S$, with the indicated value of ξ.

Integrating (4) from $\xi = 0$ to 1 gives the mean velocity related to the mass flow rate according to continuity:

$$\underbrace{\bar{w}_L = \frac{\delta^2}{3.\nu_{L.\xi=\frac{1}{4}}}\cdot\frac{\rho_L - \rho_V}{\rho_L}.g.\cos\beta - \frac{D.\delta}{8.\rho_L.\nu_{L,\xi=\frac{1}{3}}}\cdot\frac{\mathrm{d}p}{\mathrm{d}x}}_{\text{from integration}} = \underbrace{\frac{\dot{m}_L}{D.\pi.\delta.\rho_L}}_{\text{continuity}}. \quad (6)$$

When the gravitational and shear forces work in the opposite direction, $g.\cos\beta$ becomes negative. Then the velocities w_L, w_S, $\bar{w}_L$ and the condensate flow rate $\dot{m}_L$ can become negative and have the opposite direction as the vapour flow (x-axis). A negative $\bar{w}_L$ and $\dot{m}_L$ means countercurrent flow between vapour and condensate; positive values are obtained in co-current flow, which is usually the case with condensation inside tubes.

3. INTRODUCTION OF DIMENSIONLESS GROUPS

We now assume, that the differential pressure drop can be estimated by

$$\frac{\mathrm{d}p}{\mathrm{d}x} = -\zeta_V\cdot\frac{\rho_V(\bar{w}_V - w_S)^2}{2D} \quad (7)$$

where the dimensionless pressure drop is determined as for turbulent flow through a smooth tube according to Blasius (see e.g. [7], p. 219)

$$\zeta_V = 0{\cdot}3164.Re_V^{-\frac{1}{4}} \quad (8)$$

with

$$Re_V = \frac{(\bar{w}_V - w_S)D}{\nu_V}. \quad (9)$$

Further we introduce the local Reynolds number of the film according to ([7] p. 287, 281)

$$Re_L = \frac{|\dot{m}_L|}{D.\pi.\nu_L.\rho_L} \quad (10)$$

the Grashof number according to [8] (12), defined however with the tube diameter as characteristic length because the tube length does not enter our derivation.

$$Gr = \frac{D^3 . (\rho_L - \rho_V) . g . \cos \beta}{\rho_L \cdot \nu_L^2} \tag{11}$$

and finally the Nusselt number (implying a linear temperature profile in the film)

$$Nu = \frac{D}{\delta}. \tag{12}$$

Introducing (7)–(12) into (6) gives the dimensionless equation:

$$\pm 1 = \frac{\dot{m}_L}{|\dot{m}_L|} = \frac{Gr_{\xi=\frac{2}{3}}}{3 . \mathrm{Nu}^3 . Re_{L.\xi=\frac{2}{3}}} + \frac{0{\cdot}3164}{16} \times \frac{Re_V^{\frac{7}{4}}}{Nu^2 . Re_{L\cdot\xi=\frac{2}{3}}} \cdot \left(\frac{\nu_V}{\nu_{L\cdot\xi=\frac{2}{3}}}\right)^2 \cdot \frac{\rho_V}{\rho_L}. \tag{13}$$

The positive sign is for cocurrent flow of vapour and condensate, the negative sign for countercurrent flow, which can occur if the Grashof number is negative.

In the right-hand term of (13) the vapour viscosity has an overall effect of only $\nu_V^{\frac{1}{4}}$. Therefore it appears reasonable to introduce a modified vapour Reynolds number, which is defined with the viscosity of the condensate (see also Re_L of [5])

$$Re_{V,L,\xi=\frac{2}{3}} = \frac{(\bar{w}_V - w_S) . D}{\nu_{L,\xi=\frac{2}{3}}} = \frac{4\dot{m}_V}{D . \pi . \rho_V . \nu_{L,\xi=\frac{2}{3}}} - \frac{w_S . D}{\nu_{L\cdot\xi=\frac{2}{3}}}. \tag{14}$$

This Reynolds number can be broken down into two Reynolds numbers the first of which can be expressed by the vapour flow rate and the second can be determined by the following equation, derived from (5) and (7)–(12):

$$\frac{w_S . D}{\nu_{L\cdot\xi=\frac{2}{3}}} = \frac{Gr_{\xi=\frac{2}{3}}}{2 . Nu^2} + 0{\cdot}03955 \cdot \frac{(Re_{V\cdot L,\xi=\frac{2}{3}})^{\frac{7}{4}}}{Nu} \times \frac{\rho_V}{\rho_L} \cdot \left(\frac{\nu_V}{\nu_{L\cdot\xi=\frac{2}{3}}}\right)^{\frac{1}{4}} \cdot \frac{\nu_{L\cdot\xi=\frac{2}{3}}}{\nu_{L,\xi=\frac{1}{2}}}. \tag{15}$$

Equation (15) shows that the Reynolds number of the vapour can be determined only if the Nusselt number (film thickness) is known. Thus iterations are necessary.

The Reynolds number according to (14) (and (15)) is now introduced in (13). Considering the case of cocurrent flow and disappearing gravity effect ($Gr = 0$, e.g. horizontal tube) we find from (13) the Nusselt number, caused only by the vapour shear stress:

$$Nu_V = 0{\cdot}1406 \cdot \frac{(Re_{V,L,\xi=\frac{2}{3}})}{(Re_{L\cdot\xi=\frac{2}{3}})^{\frac{1}{2}}} \cdot \left(\frac{\rho_V}{\rho_L}\right)^{\frac{1}{2}} \times \left(\frac{\nu_V}{\nu_{L,\xi=\frac{2}{3}}}\right)^{\frac{1}{4}}. \tag{16}$$

We define

$$Nu_g = \left(\frac{Gr_{\xi=\frac{2}{3}}}{3 . Re_{L,\xi=\frac{2}{3}}}\right)^{\frac{1}{3}}. \tag{17}$$

This Nusselt number becomes negative when the Grashof number becomes negative. (For the heat transfer in the limiting case of negligible vapour shear forces the absolute value of Nu_g must be used, but in our further derivation the sign of Nu_g is decisive.)

With the two Nusselt numbers according to (16) and (17) the equation (13) can be simplified to:

$$\pm 1 = \left(\frac{Nu_g}{Nu}\right)^3 + \left(\frac{Nu_V}{N\pi}\right)^2 \tag{18}$$

Where again the positive sign is for cocurrent flow of vapour and condensate and the negative for countercurrent flow.

4. ANALYTICAL SOLUTION FOR THE Nu NUMBER

In order to solve (18) for Nu we introduce the variables:

$$f = \frac{Nu_g}{Nu_V}. \qquad h = \frac{Nu}{Nu_V} \tag{19}$$

which changes (18) to:

$$\pm h^3 - h = f^3. \tag{20}$$

Equation (20) is shown in Fig. 3 for cocurrent (+) and countercurrent (−) flow.

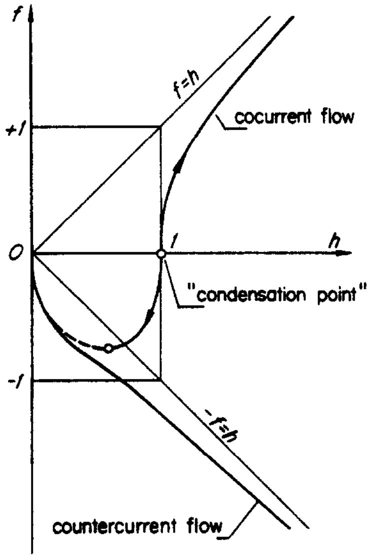

FIG. 3. f as a function of h according to (20).

If $f > 0$ shear and gravity forces work in the same direction and only cocurrent flow is possible. If $f < 0$ both forces work in the opposite direction. Above the minimum of the cocurrent function both cocurrent (two solutions) and countercurrent flow is possible, below the minimum only counter current flow.

If $f = 0$ only the shear forces of the vapour are decisive and if $f = \pm\infty$ only gravity forces are important.

We now consider the condensation along the vapour (and condensate) flow path with co-current flow.

Equations (16), (17) and (19) show that $f \sim Gr^{\frac{1}{3}} . Re_L^{\frac{1}{3}} . Re_{V \cdot L}^{1}$ and thus $|f|$ increases in flow direction, because the condensate increases and the vapour decreases. If we go back to the point where condensation begins we find that there $Nu = \infty$ (film thickness equals zero) and $f = 0$ ($Re_L = 0$ and $Re_{V,L} \neq 0$). This "condensation point" has the co-ordinates $h = 1$ and $f = 0$. From this point we move either to increasing values of $f(Gr > 0)$ or to decreasing values of $f(Gr < 0)$ down to the minimum. We never can reach the curve to the left of the minimum, drawn by a dotted line, and as this solution does not represent a physical reality in film *condensation*, it has not to be taken into account.

Now (20) can be solved for h as a function of f in the known manner (see e.g. [9], p. 63), which leads to the following equations

Countercurrent flow

$f \leqslant 0$

$$h = \left[\left(\frac{f^6}{4} + \frac{1}{27}\right)^{\frac{1}{2}} - \frac{f^3}{2}\right]^{\frac{1}{3}} - \left[\left(\frac{f^6}{4} + \frac{1}{27}\right)^{\frac{1}{2}} + \frac{f^3}{2}\right]^{\frac{1}{3}} \qquad (21)$$

Cocurrent flow

$$f \geqslant a = \frac{2^{\frac{1}{3}}}{3^{\frac{1}{2}}} \approx 0.7274$$

$$h = \left[\frac{f^3}{2} + \left(\frac{f^6}{4} - \frac{1}{27}\right)^{\frac{1}{2}}\right]^{\frac{1}{3}} + \left[\frac{f^3}{2} - \left(\frac{f^6}{4} - \frac{1}{27}\right)^{\frac{1}{2}}\right]^{\frac{1}{3}} \qquad (22)$$

$$|f| \leqslant a$$

$$h = \tfrac{2}{3}\sqrt{3} \, . \cos\left\{\tfrac{1}{3} \arccos\left(f^3 \, . \tfrac{3}{2}\sqrt{3}\right)\right\} \qquad (23)$$

and for $f < -a$ cocurrent flow between vapour and condensate is impossible.

Thus the local Nusselt number can be calculated for cocurrent and counter current flow taking into account the variation of condensate viscosity with temperature.

First Nu_V and Nu_g have to be determined according to (16) and (17), respectively, using the dimensionless groups according to (10), (11), (14) and (15) and an estimated value of Nu. Then f can be calculated according to (19) and h can be determined from f through (21), (22) or (23). Finally Nu can be calculated from h and Nu_V according to (19). With this value of Nu the calculation can be repeated, until the estimated and calculated Nu-number agree.

The critical Reynolds number of the film up to which laminar film flow can be assumed, is difficult to predict with combined gravity and shear effects. According to [4] and [7] it should be $60 \leqslant Re_{L,C} \leqslant 350$. The lower value ([14]) is for the case of decisive shear effects, the higher value ([7]) for negligible shear effects.

5. SIMPLE APPROXIMATING EQUATIONS

For hand calculations the presented method may be slightly inconvenient, especially (21)–(23). Therefore simple approximating equations are derived which yield the Nusselt number directly as a function of the dimensionless groups Re_L, $Re_{V,L}$, Gr, ρ_L/ρ_V and ν_L/ν_V.

Instead of (21)–(23) the following approximating functions for h as function of f can be used in the given range of f when relative errors of h up to 0·5 per cent are tolerated. They cover only the range of f which is of practical importance.

Cocurrent (+) and countercurrent (−) flow

$$f \geqslant 1{\cdot}1 \quad \text{cocurrent flow} \qquad f \leqslant -1{\cdot}25 \quad \text{countercurrent flow}$$

$$h = \pm f + \frac{1}{3f} \tag{24}$$

Cocurrent flow

$$1{\cdot}3 \geqslant f \geqslant 0{\cdot}5$$

$$h = 0{\cdot}974 + 0{\cdot}346\,.f^2 \tag{25}$$

$$|f| \leqslant 0,5$$

$$h = 0{\cdot}998 + 0{\cdot}505\,.f^3. \tag{26}$$

Substituting f in (24)–(26) according to (19), (17) and (16) and ignoring the different reference temperatures in (16) and (17) leads to the following practicable equations:

With dimensionless group $A(A = f/4.9306)$ for the determination of the validity range

$$A = \frac{Gr^{\frac{1}{3}}\,.\,Re_L^{\frac{1}{2}}}{Re_{V,L}^{\frac{7}{8}}} \cdot \left(\frac{\rho_L}{\rho_V}\right)^{\frac{1}{2}} \cdot \left(\frac{\nu_L}{\nu_V}\right)^{\frac{1}{8}} \tag{27}$$

which could be evaluated to any reasonable reference temperature, e.g. T_W or T_S, or any estimated value near these temperatures, we find for *cocurrent (+) and countercurrent (−) flow* in the range of

$$A \geqslant 0{\cdot}22 \quad \text{cocurrent flow} \qquad A \leqslant -0{\cdot}26 \quad \text{countercurrent flow}$$

$$Nu = \pm\left(\frac{Gr}{3\,.\,Re_L}\right)^{\frac{1}{3}} + 0{\cdot}00951 \times \frac{Re_{V,L}^{\frac{7}{4}}}{Gr^{\frac{1}{3}}\,.\,Re_L^{\frac{2}{3}}} \cdot \frac{\rho_V}{\rho_L} \cdot \left(\frac{\nu_V}{\nu_L}\right)^{\frac{1}{4}} \tag{28}$$

where all properties should reasonably be determined at the reference temperature $T_v = \frac{3}{4}T_W + \frac{1}{4}T_S$, because Nu is close to $|Nu_g|$. Further we find for *cocurrent flow* in the range of:

$$0{\cdot}26 \geqslant A \geqslant 0{\cdot}1$$

$$Nu = 0{\cdot}137 \cdot \frac{Re_{V,L}^{\frac{7}{8}}}{Re_L^{\frac{1}{2}}} \cdot \left(\frac{\rho_V}{\rho_L}\right)^{\frac{1}{2}} \cdot \left(\frac{\nu_V}{\nu_L}\right)^{\frac{1}{8}} + 1{\cdot}183 \times \frac{Gr^{\frac{2}{3}}}{Re_L^{\frac{1}{6}}\,.\,Re_{V,L}^{\frac{7}{8}}} \cdot \left(\frac{\rho_L}{\rho_V}\right)^{\frac{1}{2}} \cdot \left(\frac{\nu_L}{\nu_V}\right)^{\frac{1}{8}} \tag{29}$$

with the mean reference temperature (between $\frac{3}{4}T_W$ and $\frac{2}{3}T_W$) $T_v = 0{\cdot}71.\,T_W + 0{\cdot}29.\,T_S$, because both Nu_g and Nu_v contribute about equally.

And in the range of

$$|A| \leqslant 0{\cdot}1$$

$$Nu = 0{\cdot}140 \cdot \frac{Re_{V,L}^{\frac{7}{8}}}{Re_L^{\frac{1}{2}}} \cdot \left(\frac{\rho_V}{\rho_L}\right)^{\frac{1}{2}} \cdot \left(\frac{\nu_V}{\nu_L}\right)^{\frac{1}{8}} + 8{\cdot}51 \times \frac{Gr}{Re_{V,L}^{\frac{7}{4}}} \cdot \frac{\rho_L}{\rho_V} \cdot \left(\frac{\nu_L}{\nu_V}\right)^{\frac{1}{4}} \tag{30}$$

with the reference temperature $T_v = \frac{2}{3}T_W + \frac{1}{3}.\,T_S$ according to (16), because Nu is close to Nu_V.

Thus the Nusselt number can be calculated directly from the dimensionless groups according to (10), (11), (14), (15) and the viscosity and density ratios. $Re_{V,L}$ can be determined iteratively or, especially in the range of (28) where gravity effects are decisive, it may be determined by introducing the simplification that $Nu = \infty$ or $W_S = 0$.

Equations (28)–(30) (as well as the analytical solution of section 4), derived for local heat transfer, can be used as an approximation for mean Nu numbers, provided that the dimensionless groups are now determined with *mean* values of temperature, condensate flow rate and vapour flow rate (e.g. arithmetic mean values).

6. THE ADIABATIC MIXING TEMPERATURE OF THE FILM

For the determination of the enthalpy flow of the condensate film one needs its adiabatic mixing temperature, usually derived with the assumption of a constant heat capacity of the fluid. If for simplification we assume also a linear temperature profile in the film, then the dimensionless adiabatic mixing temperature M can be derived according to (28) and (30) of [6] by this integration:

$$M = \frac{\bar{T}_L - T_W}{T_S - T_W} = \int_{\xi=0}^{\xi=1} \frac{w_L}{\bar{w}_L}\cdot(1-\xi)\,d\xi. \tag{31}$$

Simplifying further we neglect the temperature dependence of the viscosity (and the other properties). Thus substituting in (31) w_L according to (4) with $\nu_S = \nu_W = \nu_L$ and $\bar{w}_L$ according to (6) leaving out the indices $\xi = \frac{2}{3}$ and $\xi = \frac{3}{4}$ and integrating gives:

$$M = \frac{\frac{5}{24} - \frac{1}{12}Q}{\frac{1}{3} - \frac{1}{8}Q} \tag{32}$$

$$Q = \frac{D}{\delta(\rho_L - \rho_V).g.\cos\beta}\cdot\frac{dp}{dx}. \tag{33}$$

Introducing into (33) the dimensionless groups according to (7)–(9), (11) and (12) and substituting Q in (32) gives:

$$M = \frac{5}{8}\cdot\frac{1 + \frac{16}{15}\cdot N}{1 + N} \tag{34}$$

where

$$N = 0{\cdot}0593\cdot\frac{Re_{V,L}^{\frac{7}{4}}.Nu}{Gr}\cdot\frac{\rho_V}{\rho_L}\cdot\left(\frac{\nu_V}{\nu_L}\right)^{\frac{1}{4}}. \tag{35}$$

Equations (34) and (35) should be used, when employing the approximating equations (28), (29) or (30).

The dimensionless groups in (35) should be determined all at the same reference temperature, prescribed by the Nu equation. However, the liquid viscosities appearing in (35) cancel, and thus they could be determined at any common reference temperature.

The dimensionless group N can be positive or negative according to the sign of Gr. If $Gr \to 0$ (horizontal tube) or $Re_{V,L} \to \infty$ (very high vapour velocity) $N \to \infty$ and thus $M \to \frac{2}{3}$. If $Re_{V,L} \to 0$ (no vapour shear) $N \to 0$ and $M \to \frac{5}{8}$, the known value shown in [6]. For $Gr \geqslant 0$ $\frac{2}{3} \leqslant M \leqslant \frac{5}{8}$. For $Gr < 0$ any limiting values are difficult to predict with (34) and (35), but the problem is overcome with the following derivation:

Comparing (35) with (16) and (17) gives, with (19):

$$N = \frac{Nu.Nu_V^2}{Nu_g^3} = \frac{h}{f^3}. \tag{36}$$

Substituting f^3 in (36) according to (20) and introducing N in (34) leads to the very simple equation:

$$M = \frac{5}{8}\cdot\left(1 \pm \frac{1}{15.h^2}\right) \tag{37}$$

where the positive sign is for cocurrent flow, the negative for countercurrent flow.

With $h \to 0$ $M \to \pm\infty$ which means physically that the condensate mass flow rate $\dot{m}_L \to 0$. This is reasonable in the point $f = 0$, $h = 0$ because here is the transition between cocurrent and countercurrent flow or between positive and negative $\dot{m}_L$, respectively. At this point $\dot{m}_L = 0$, however, the resulting enthalpy flow is not zero because the velocity and temperature profiles are not flat. Therefore the adiabatic mixing temperature must become infinite. Thus for countercurrent flow $-\infty \leqslant M \leqslant \frac{5}{8}$. At the minimum of the cocurrent curve in Fig. 3, where $h = (\frac{1}{3})^{\frac{1}{2}}$ the dimensionless adiabatic mixing temperature becomes $M = \frac{3}{4}$. Thus for cocurrent flow $\frac{3}{4} \geqslant M \geqslant \frac{5}{8}$.

Equation (37) should be used, when employing the analytical solution according to section 4.

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CONDENSATION EN FILM LAMINAIRE DANS DES TUBES; CALCUL DE LA RESISTANCE ET LA TEMPERATURE DE MELANGE ADIABATIQUE LOCALES

Résumé—L'épaisseur et la température de mélange adiabatique ont été calculées localement par rapport à l'action combinée de la gravitation et des forces de cisaillement entre la vapeur en écoulement turbulent et le condensat en écoulement laminaire (à co-courant ou à contrecourant).

Afin d'obtenir un caicul précis de l'épaisseur du film, le profil de viscosité a été pris en considération. Il en a résulté trois températures de référence pour la viscosité.

Pour le calcul de la température de mélange adiabatique locale ont été adoptées des propriétés locales constantes du condensat ainsi qu'un profil linéaire de température. Compte tenu de l'action combinée de la gravitation et des forces de cisaillement, la température adiabatique locale n'est donc plus une valeur constante (pour des températures de paroi et de surface de film données), mais la fonction de nombreuses variables.

A partir de la solution analytique (itérative) ont été dérivées des équations approximatives pour la résistance locale du film, ainsi facilitant le calcul à main.

LAMINARE FILMKONDENSATION IN ROHREN: BERECHNUNG DES ÖRTLICHEN FILMWIDERSTANDES UND DER ÖRTLICHEN ADIABATEN MISCHTEMPERATUR

Zusammenfassung—Die örtliche Dicke und adiabate Mischtemperatur des Kondensatfilms wurde für den Fall berechnet, dass gleichzeitig Schwer- und Schubkräfte wirken zwischen turbulent strömendem Dampf und laminar strömendem Kondensat bei Gleich- oder Gegenstrom.

Für eine möglichst genaue Berechnung der Filmdicke wurde das Zähigkeitsprofil im Kondensat mitberücksichtigt. Hierbei ergaben sich drei verschiedene Bezugstemperaturen für die Zähigkeit.

Bei der Berechnung der örtlichen adiabaten Mischtemperatur wurden örtlich konstante Stoffwerte und ein lineares Temperaturprofil im Kondensat angenommen. Die örtliche adiabate Mischtemperatur ist nicht mehr konstant (bei festliegender Wand- und Filmoberflächentemperatur) sondern eine Funktion von vielen Veränderlichen, wenn gleichzeitig wirkende Schub- und Schwerkräfte berücksichtigt werden.

Aus der analytischen Lösung (iterativ) wurden Näherungsgleichungen für den örtlichen Filmwiderstand entwickelt, die für Handrechnungen besser geeignet sind.

ЛАМИНАРНАЯ ПЛЕНОЧНАЯ КОНДЕНСАЦИЯ В ТРУБАХ. РАСЧЕТ ЛОКАЛЬНОГО ПЛЕНОЧНОГО СОПРОТИВЛЕНИЯ И ЛОКАЛЬНОЙ АДИАБАТИЧЕСКОЙ ТЕМПЕРАТУРЫ ПЕРЕМЕШИВАНИЯ

Аннотация—Рассчитывались локальная толщина пленки и адиабатическая температура перемешивания в случае совместного действия сли тяжести и сдвига между турбулентным потоком пара и ламинарным потоком конденсата (спутный и встречный потоки).

При точном расчете толщины пленки учитывался профиль вязкости конденсата, в результате чего для вязкости получены три различные исходные температуры.

При расчете локальной адиабатической температуры смешения локальные свойства конденсата принимались постоянными, а его температурный профиль линейным. Тогда при учете сил тяжести и сдвига локальная температура смешения перестает быть постоянной (при заданной локальной температуре стенки и поверхности пленки) и становится функцией многих переменных.

На основе аналитического итерационного решения получены приближенные уравнения для локального сопротивления пленки, облегчающие «ручной» счет.